

# Einstein Summation Convention

Einstein's Summation Convention (ESO) is just a simple way of compressing really large equations into a compact form. A Large equation like the one below becomes as simple as:

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n = a_i x_i$$

So,

normal summation symbol

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n = \sum_{i=1}^n a_i x_i = a_i x_i \quad \sim \text{We just drop the } \sum \text{ sum notation}$$

$a_i x_i$  means

There are 3 simple rules

Twice repeated index MEANS sum in the same equation

No index may occur more than twice in an expression

Free index shall have same range as dummy index

$$a_{ii} x_k = a_{11} x_k + a_{22} x_k + \dots + a_{nn} x_k$$

$$a_{ij} x_j = a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n$$

row      column

$y_i = a_{ir} x_r$  by arranging the indices in a special way we can build a system of equations, matrix algebra!

$$\left. \begin{aligned} y_1 &= a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \\ y_2 &= a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \\ y_3 &= a_{31} x_1 + a_{32} x_2 + a_{33} x_3 \end{aligned} \right\} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

This is very convenient the fact that  $y_i = a_{ir} x_r$  is the same as

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

The main problem is getting used to this new way of looking at things

## Double Sums

$$a_{ij} x_i y_j \rightarrow n^2 \text{ terms} \quad i, j \rightarrow n$$

$$\hookrightarrow a_{1j} x_1 y_j + a_{2j} x_2 y_j + a_{3j} x_3 y_j$$

$$a_{11} x_1 y_1 + a_{12} x_1 y_2 + a_{13} x_1 y_3 + a_{21} x_2 y_1 + a_{22} x_2 y_2 + a_{23} x_2 y_3$$

$$+ a_{31} x_3 y_1 + a_{32} x_3 y_2 + a_{33} x_3 y_3$$

$$y_i = c_i^r a_{rs} x_s \quad \text{let } n=2$$

$$\begin{aligned} y_1 &= c_1^1 a_{11} x_1 + c_1^2 a_{21} x_2 + c_1^1 a_{12} x_1 + c_1^2 a_{22} x_2 \\ y_2 &= c_2^1 a_{11} x_1 + c_2^2 a_{21} x_2 + c_2^1 a_{12} x_1 + c_2^2 a_{22} x_2 \end{aligned}$$